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By A. D. Schwope and L. R. Jackson
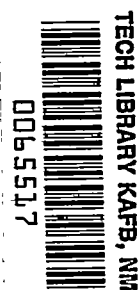
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SUMMARY

This report summarizes numerous theories relating to creep phenomena and the extent of current knowledge on the subject. Various possible mechanisms by which creep occurs both in single crystals and in poly-crystals are described. An extensive bibliography of published works in this field is also included.

INTRODUCTION

At present, there is a great deal of interest in the behavior of metals and alloys under stress at high temperatures. This interest is stimulated not only by the necessity for designing aircraft structures to operate at high temperatures, but also by the desire to develop more effective alloys for high-temperature service.

Experimental data available are not sufficiently detailed nor do they cover a wide-enough range in conditions to be effective for use in design. While there is considerable activity in the development of theories to account for creep behavior, these developments are not com-plete enough to be of much help.

The purpose of this survey is to determine what types of data are available and to review theories of creep with the idea of determining an effective experimental program on the fundamentals of creep which would fit current needs. The body of the report presents a review of work on creep. References 1 to 199 constitute a bibliography of many of the published works in this field.

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FUNDAMENTAL CREEP PHENOMENON IN METALS

The phenomenon of time-dependent distortion of metals under stress has been recognized for over a hundred years. This deformation originally described as tenacity has come to be known as creep. Although the majority of creep tests are run at elevated temperatures, some metals, such as lead and tin, exhibit considerable deformation at room temperature.

The knowledge of the characteristics of various metals at elevated temperatures is essential for the design of equipment required to operate at these temperatures. The determination of the creep properties of various metals and alloys requires the carrying out of tests over relatively long periods of time (1000 to 10,000 hr) at carefully controlled temperatures using very sensitive strain-measuring devices. These tests have proved to be a powerful tool in evaluating materials for high-temperature service. Of equal importance is the development of new alloys for higher stresses and temperatures. This study consists largely of cut-and-try experiments.

It is believed that if more were known about the mechanism of creep this knowledge could be profitably applied in the development of new temperature-resistant alloys. Furthermore, a better understanding of creep phenomena will allow more efficient use of presently available materials in design. It is with this purpose in mind that this report is written. It will serve as a review of existing knowledge of the mechanism of creep.

This review is divided into two main sections. These cover single-crystal and polycrystalline metals. The modes of deformation in short-time or low-temperature tests are considered along with the atomistic and phenomological views of the behavior of metals during creep.

CHEMICAL-RATE THEORY

Since creep is a rate process and much of the theoretical work has been accomplished using chemical-rate theory as a basis, it is appropriate that this theory be reviewed. It is well-known in chemistry that a system may exist in a metastable equilibrium. For example, hydrogen and oxygen can coexist at room temperature without reacting to form water. In order for the reaction to proceed, energy must be added, and the

system must pass over an energy hump of unstable equilibrium to the more stable state. This process is shown in the following sketch:

Course of reaction
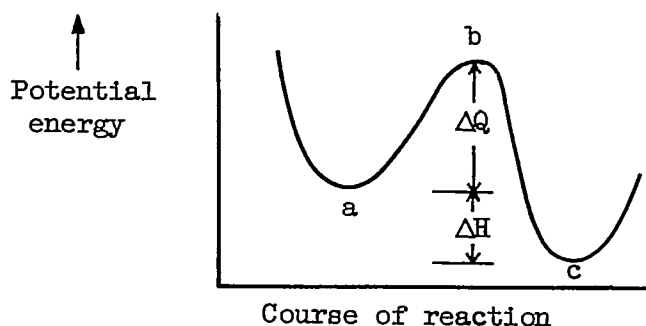
Unless the reactants at a can acquire enough energy to pass over the energy hump at b, no reaction can take place. This additional energy is known as the activation energy. The absorption of energy ΔH, once the hump is passed and the reaction proceeds without energy being added, is known as the heat of reaction.

The rate at which the reaction a $\longrightarrow$ b proceeds depends on the height of the potential-energy hump, and the number of complexes which possess enough energy to pass over the hump. The first condition is self-explanatory; the smaller ΔQ is, the more rapidly the reaction can take place. The second condition depends on the distribution of the energy in a system. The number of complexes possessing the required energy can be calculated using the Maxwell-Boltzman distribution law, which states that the number having an energy greater than ΔQ is proportional to

$$e^{-\Delta Q/RT} \tag{1}$$

where R is the molar gas constant and T is the absolute temperature in ^{O}K.

The rate of the reaction can then be expressed by the Arrhenius equation

$$V = Ae^{-E/RT} \tag{2}$$

where E is the difference in heat content between the activated and initial states.

Eyring and other investigators have shown that the rate of reaction is more closely described using the free energy of activation ΔF, and

have developed a more exact theory. Utilizing statistical mechanics,
it can be shown that the number N of activated complexes passing a
point P multiplied by their rate is given by

$$N = \frac{kT}{h} e^{-\Delta F/RT}$$

$$= Ae^{-\Delta F/RT} \tag{3}$$

where

ΔF free energy of activation per gram molecule

k Boltzman's constant, ergs per degree

h Planck's constant, ergs per second

From thermodynamics it can be shown that

$$\Delta F = \Delta Q - T \Delta S \tag{4}$$

where ΔS is the entropy of activation, and equation (3) becomes

$$N = \frac{kT}{h} e^{-\Delta Q/RT} e^{\Delta S/R} \tag{5}$$

Equation (5) is then the expression for the frequency of jumps in the
forward and backward directions, which must be equal for equilibrium
conditions.

If some outside shear force f is applied to the system, it will
act in such a way as to tip the potential-energy curve in the direction
of the force. (See fig. 1.) The applied force acting on a single funda-
mental unit is equal to fA' where A' is the effective area per unit
on the shear plane. The energy that the moving unit acquires when it
has reached the activated state is then equal to flA', where l is the
distance through which the force acts. The net rate of flow N' in the
forward direction is then given by

$$N' = \frac{kT}{h} e^{-\Delta F/RT} \left(e^{fAl/kT} - e^{-fAl/kT} \right) \tag{6}$$

or

$$N' = \frac{kT}{h} e^{-\Delta F/RT} \sinh \frac{fA\lambda}{kT} \tag{6a}$$

If $fA\lambda \ll kT$ then

$$N' = \frac{kT}{h} e^{-\Delta F/RT} \frac{A\lambda f}{kT}$$

$$= \frac{A\lambda f e^{-\Delta F/RT}}{h} \tag{7}$$

If $fA\lambda \gg kT$ then equation (6) becomes

$$N' = \frac{kT}{h} e^{-\Delta F/RT} e^{fA\lambda/kT} \tag{8}$$

Equations (3), (4), (6a), (7), and (8) have become the foundation for modern creep analysis. Most of the investigators noting the similarity between certain creep phenomena and a chemical reaction have either applied these equations directly or have attempted to modify them to fit the experimental data. Much of the application of the chemical-rate theory has been centered about the secondary creep-rate stage of polycrystalline metals. Table 1 lists some of the various theories and the equations to which they correspond.

It is seen that, since the original theory as developed by Eyring described the viscosity of liquids, these later equations are decidedly empirical and are not based on any fundamental criterions.

DEVELOPMENT OF CREEP CURVES

The presentation of the results of creep tests is normally accomplished by plotting curves of extension against time using rectangular coordinates. Stress and temperature are held constant for each curve. For the various ranges of temperature and stress, the material can vary widely in its reaction. If the stress is low enough, the short-time yield strength may not be exceeded and the reaction may be largely

elastic (fig. 2(a)). With increasing stress, the yield strength is exceeded and plastic flow may result. This may or may not be accompanied by strain-hardening (fig. 2(b)). If the temperature is high enough, the plastic flow may be viscous and the rate of flow is proportional to the stress (fig. 2(c)). While it might seem most logical to begin with a description of creep in single crystals, there are so many more data on polycrystalline materials that these will be discussed first.

Many of the creep curves can be simply divided into two components, the viscous or steady state and the transient. This division is shown in figure 3 along with the sum of the two components. The nature of these types of flow and their cause will be discussed in later sections.

The correlation of creep data has been made extremely difficult by the many types of tests being used. From the fundamental viewpoint, the two types of tests of interest at this time are the constant-load and constant-stress tests. The constant-load test is one in which the initial load is calculated to produce a desired stress based on the original area. As the test proceeds and the specimen reduces in area, the stress is automatically proportionally increased. In many cases, the reduction in area is slight and the increase is negligible, but when it is considered that an increase in stress of 2 percent may cause a 50-percent increase in creep rate, it is evident that this factor should not be overlooked. The constant-stress test is one in which the load is reduced during the test to compensate for the area reduction. The load is usually reduced by assuming that Poisson's ratio is equal to 0.5, and, in the initial stages of creep, it is very doubtful whether this is true. The contrast between the constant-stress and constant-load test is shown in figure 4, which shows data for lead at 15° C taken from reference 3.

The constant-load test is commonly used by engineers and is divided into three stages as indicated in figure 4. The initial stage (0-1) includes the immediate or elastic deflection and the portion of the curve showing a decelerating rate of creep. In many cases this resembles the transient curve but is most likely a combination of the viscous and transient stages. The second or steady-rate stage (1-2) is characterized by the elongation-time curve becoming linear. However, in some instances, this does not happen and the third stage is immediately entered. The second stage is then defined as a point when the creep rate reaches a minimum. This stage is usually found in polycrystalline metals where grain-boundary flow is predominant. The third or final stage (2-3) is characterized by a rapidly accelerating creep rate ending in fracture. As will be discussed later, some investigators are of the opinion that the inflection point can be related to the strain at maximum load in a constant-strain-rate tension test.

It is interesting to derive the creep curve from a short-time constant-strain-rate tension test. The usual tension test is run at a constant strain rate and the calculated nominal stress s is commonly found to be a function of strain rate $\dot{\epsilon}$ and strain ϵ at constant temperatures such that $F(\epsilon,\dot{\epsilon},s) = 0$. The stress-strain curves form a family as shown in figure 5(a). The curve is displaced upward and a greater uniform strain is obtained as the strain rate increases.

The ordinary long-time creep tests at constant load can be considered to be a similar function of s, ϵ, and $\dot{\epsilon}$. A simple way of graphically determining the creep curve from constant-strain-rate tests has been indicated by Nadai (reference 148) and later by Hollomon (reference 104). For each of the loads, the curve of strain against strain rate can be determined by a set of horizontal planes (s = Constant). These lines show how $\dot{\epsilon}$ changes with ϵ (fig. 5(b)). The minimum creep rate occurs at lower strains as the stress decreases. From curves of figure 5(b), elongation-time curves can be constructed at constant loads (fig. 5(c)). The familiar shape of the curves is at once apparent. The initiation of the third stage is caused not by an unbalance of strain-hardening and softening but simply by the fact that creep has reached the elongation at maximum load. Thus, if one assumes that $F(s,\epsilon,\dot{\epsilon}) = 0$, the constant-load curves can be derived from constant-strain-rate tests.

It should be pointed out, however, that Dorn and Lietz (reference 62a) have recently obtained data on aluminum alloy 3S-H12 which do not agree with the argument presented above. Thus, it would seem that further experimental work is required before these principles can be applied with confidence.

It is evident that as the specimen creeps its cross-sectional area must decrease and the actual or true stress increase. The difference between the true and nominal stresses is a direct function of the strain; that is, if the specimen elongates 2 percent in a tensile creep test its true stress is 2 percent higher than the nominal stress. To compare constant-strain-rate tests with constant-stress tests, the former must be plotted as a function of the true stress. Theoretically, the strain should be the true or logarithmic strain δ given by the equation

$$\delta = \log_e(1 + \epsilon) \qquad (9)$$

and the true strain rate $\dot{\delta}$ is then given by

$$\dot{\delta} = \frac{\dot{\epsilon}}{1 + \epsilon} \qquad (10)$$

The true stress, which is the load divided by the instantaneous area, can be converted from nominal stress by

$$\sigma = s(1 + \epsilon) \tag{11}$$

The true stress-strain curve for various constant strain rates is shown in figure 6(a). Note the absence of any maximum stress point. The plot of strain rate against strain is then arrived at as before and is shown in figure 6(b). There is no longer the characteristic minimum at maximum elongation and the curves have a decreasing slope eventually reaching a constant value. The corresponding strain-time curves are shown in figure 6(c). The point of inflection or initiation of the so-called third-stage creep has disappeared and a constant rate of creep becomes the final stage.

These representations based on the assumption that $F(\sigma, \delta, \dot{\delta}) = 0$ look promising, but real metals fail to follow them exactly. Often in low-temperature tests the creep strain slowly reaches some limiting value resulting in a constantly decreasing creep rate. Also, the absence of the third or rapidly accelerating stage in constant-stress tests has been questioned and recently data have been advanced showing that a compression creep test exhibits a third stage (reference 180). Thus, the validity of the assumption that $F(\sigma, \delta, \dot{\delta}) = 0$ is also open to question.

SINGLE CRYSTALS

Plastic Flow

A metal crystal under a uniaxial tensile force initially deforms an amount proportional to the stress. This deformation, commonly termed elastic, is reversible if the force is removed. If the force or stress exceeds a certain limit, permanent or plastic deformation sets in. The point of departure of the stress-strain curve from a straight line will be termed the yield point. Some investigators have indicated that the yield point becomes lower as the sensitivity of the extensometer increases. However, Chalmers (references 41 and 42), using a very precise extensometer system, reports a definite point at which elasticity disappears for tin single crystals, and states that higher degrees of accuracy would not reduce this point.

Increases in temperature would be expected to reduce the yield point. This is found to be true if the temperature is above the so-called recrystallization range. Below this temperature range there is a tendency in some crystals to increase the elastic limit. Miller

(reference 139) found that the elastic limit in pure silver showed an increase up to 300° C. Dr. A. V. de Forest, in a discussion of this paper, pointed out that this might be due to either age- or precipitation-hardening.

The stress required for the initiation of plastic flow in single crystals has been termed the critical shearing stress. When this stress is reached in a favorably oriented crystal, slip takes place. Slip is the movement of one part of the crystal relative to another along a definite plane in a definite direction. It has been determined that the critical shearing stress is the lowest in the most closely packed plane and in the direction which contains the most atoms. During deformation, as slip occurs along the favorable planes, it is found that these slip bands are parallel to one another and a definite distance apart (about 1 micron).

From known atomic constants, the critical shearing stress can be calculated fairly easily. However, it has been found experimentally that slip takes place in real crystals at stresses of the order of one-thousandth of the calculated value. The most logical reason advanced for this irregularity is that a metal crystal contains a great many imperfections which act as stress raisers or nucleating points for the slip to begin.

There have been two outstanding theories of the mechanism of slip; the first proposed by Becker in 1925 utilizes thermal oscillations while the newer dislocation theory postulated a definite imperfection and mechanisms for its growth and multiplication. It is worth while to review these two theories, for they form the basis for much of the fundamental work on the creep process.

The Becker theory (reference 164) states that the stress at any point is not constant but fluctuates because of thermal oscillations. These local fluctuations have a possibility of becoming large enough to allow small groups of atoms to pass one another and so result in a slip process. It is probable that this process is always taking place but without the addition of an external stress the groups of atoms cannot become large enough to become a stable unit. The relations can be expressed algebraically by considering that the thermal fluctuations must provide a stress equal to $\tau_0 - \tau$, where τ_0 is the net stress necessary for glide and τ the applied shear stress. If there are n regions of volume V on which this stress acts, the shear-strain rate γ is then given by

$$\dot{\gamma} = \frac{n\gamma_0}{t} \exp\left[\frac{-V(\tau_0 - \tau)^2}{2GkT}\right] \tag{12}$$

where

n   number of regions of volume V

γ_0 unit shear strain in volume V

t   time for one volume to slip past one other

G   shear modulus

k   Boltzman's constant

T   absolute temperature

If tests could be run at absolute zero, τ_0 would be found to be the value of the critical shear stress. However, it has been found that τ_0 is of the same order as τ at room temperature so that the energy imparted by thermal fluctuations cannot be significant. Therefore, the deformation must be confined to regions of high stress concentration.

The second important mechanism for slip of single crystals is the movement of dislocations. This relatively new theory has gained wide popularity and is described in detail by many authors. (See references 114, 152, 186, and 143.) A dislocation is the result of an imperfection in the lattice network which causes a misfit of atoms. The best-known type is the edge or line dislocation. Some properties of the edge dislocation are: (1) Dislocations on one plane are of two signs, positive and negative. A positive dislocation is one in which material above the dislocation is in compression and that below it is in tension. (2) If two dislocations of unlike sign meet, they annihilate each other. (3) Dislocations of like sign repel each other and those of unlike sign attract each other. (4) A dislocation will move if the applied shear stress exceeds some definite value.

It is calculated that the energy of the dislocation is about 1 to 5 electron volts per atomic plane. On this basis, it has been postulated that any crystal containing dislocations is thermodynamically metastable. This quality of energy cannot be generated by thermal fluctuation. The possibility, however, of the energy being lowered in the presence of an applied stress is good, although no evidence of the mechanism has been found.

It is not certain whether the dislocations are formed during stressing or whether they are already present. If they are formed, they are formed by a process of nucleation and growth. This, conceivably, could happen with the aid of thermal fluctuations with the applied stress

guiding the direction of growth. Brag (references 31 and 122) and his
associates have recently indicated, using his famous soap-bubble model,
that it is unlikely that stresses cause a metal to slip by creating dis-
locations. The thermal fluctuation energy kT is about one-hundredth
of the estimated energy of a dislocation, so that even the joint action
of stress and thermal fluctuation seems a doubtful source of dislocation
in a perfect lattice.

If an otherwise perfect crystal contains only a few dislocations,
it is likely that these will move through the crystal with the speed of
sound generating other dislocations. The yield point of a metal is then
a measure of the resistance to movement of the dislocation. It can be
shown how the phenomena of double yield points and strain-hardening are
explained on this basis. These will not be considered further at this
time; however, strain-hardening will be taken up later in this report.

The dislocation theory has become very popular, partly because of
its ability to be altered to fit any situation. Where the picture of
edge dislocations has failed, a new type, the screw dislocation, has been
invented. Although dislocations have never been seen, the results
attributed to dislocations are quite evident and thus lend credence to
the theory.

Experimental Observations on Transient Creep

Creep tests using single crystals have been relatively few in
number and usually involve time periods of a day or less. The type of
creep associated with short times is usually the transient component.

Chalmers (references 41 and 42), using single crystals of tin of
two different degrees of impurity, found the initial creep rate to be a
direct function of the stresses for very low stresses (< 100 g/sq mm).
This would indicate that a single crystal behaved in a viscous manner
following equation (7). Since these stresses are below the critical
shearing stress, some mechanism other than slip must be operative. For
larger stresses, the creep rate increased markedly. Chalmers arbitrarily
designated the linear range as α and the increasing-rate range as β.
The α range has been commonly termed microcreep and the β range
macrocreep. The β range is characterized by a limiting stress which
produces a very rapid extension.

Burghoff and Mathewson studied creep of single crystals of 70-
30 brass at a number of temperatures (reference 35). Their results
indicate a mechanism similar to that of a chemical reaction. At first
only a few slip planes are available for flow; as slip progresses new
slip planes are made available at an ever-increasing rate. If the

temperature is low (fig. 7), the metal strain-hardens and creep eventually stops. If a higher test temperature (fig. 8) is used, the crystal continues to slip indefinitely. It should be noted that there is a great difference in the total deformation in these two tests, so that a direct comparison is not absolutely valid. It does appear though that at the higher temperature any stress, no matter how small, will cause creep to take place.

Gensamer and Mehl (reference 78), using constant-load tests on single crystals of iron, found a well-defined yield point at room temperature. This is contrary to Chalmers' results on tin which indicated creep at any stress. A typical curve from Gensamer and Mehl is shown in figure 9 for a stress above the yield point. This curve is similar to those of figures 7 and 8 in that an induction period is present initially. Their explanation of the shape of the curve was based on concepts of nucleation and growth of strained regions. Smith and Beck in a discussion of this paper pointed out that a similar induction period was found by them in crystals of zinc. They presented Orowan's interpretation of slip as an alternative explanation of the inductive period.

This mechanism makes use of the thermal fluctuations described in a following section on theory and the induction period is simply a waiting period for a fluctuation to increase the stress enough for slip to occur.

Hirst (reference 99), using metallographic techniques, studied creep of single crystals of lead at room temperature. Creep was found to take place initially by formation of slip bands and, ultimately, the material breaks up into small blocks with slip bands as the block boundaries. These blocks are rotated during deformation independently of one another, which results in orientations differing from those of the original crystal. These blocks decrease in size as deformation continues. Ultimately, the crystal will consist of a mass of small blocks of relatively undistorted lattice joined by regions of highly deformed material. This is similar to Gough's theory of crystallite formation (reference 86). The appearance of a third or accelerating creep-rate stage could be easily explained by the process of recrystallization of these crystallites.

The preceding observations on the creep of single crystals raises the question whether or not the work was done with a single crystal. Certainly, Chalmers' microcreep presupposes some irregularities in the crystal in order for it to act in a viscous manner. Hirst found that the crystal breaks up into small blocks upon straining. Many investigators have found evidences of a mosaic structure within a crystal. These findings indicate the possibility of micro grain boundaries, or regions of misfit between blocks or cells of slightly different orientation. If this is true, then a single crystal, after some plastic

deformation, should behave as a polycrystalline metal. The temperature and rate of deformation are important factors which need to be more fully considered in the study of single-crystal creep.

Theory of Transient Creep

A single crystal which is subjected to a stress for relatively long periods of time will deform. How fast it deforms will depend on the temperature, stress, and the amount of prestraining. For the present, a virgin crystal will be considered. If the stress and temperature are sufficiently low, the crystal will undergo an immediate elastic extension followed by a period of decelerating creep rate. In some cases after a sufficient time has elapsed the rate of creep will drop to zero and no further deformation will be observed. This type of deformation is known as transient creep.

Transient creep was initially observed in polycrystalline wires by Andrade (reference 4) but has since taken on an added significance in the light of the dislocation theory.

A simple picture using the dislocation theory is obtained by supposing that the creep is caused by the moving of dislocations. The creep rate would then be given by

$$\frac{d\epsilon}{dt} = N\overline{V}\lambda \tag{13}$$

where

N number of dislocations per unit area

$\overline{V}$ mean velocity of dislocation propagation

λ slip distance associated with passage of one dislocation

This relation meets difficulty because the required velocity must be very low for reasonable values of N.

Exhaustion creep.- Mott and Nabarro (reference 142) have advanced a very plausible theory which applies to the transient creep at low stresses and temperatures. Their name for this type of creep is exhaustion creep, which indicates that deformation is using up the available dislocations and must eventually cease. If a stress σ is applied to a metal, all dislocations for which $\sigma_i < \sigma$ will move without the aid of heat. Here, σ_i indicates the stress necessary to move a dislocation. At the end of this instantaneous deformation there will be a

number $N(\sigma_i)$ of dislocations left. If, then, the stress is increased by $d\sigma_i$ to $\sigma_i + d\sigma_i$, the number of dislocations able to move are $N(\sigma_i)d\sigma_i$. It is assumed that during creep the hardening is due to exhaustion of dislocations and $N(\sigma_i)$ does not change appreciably.

A dislocation for which $\sigma_i > \sigma$ will move only if it receives additional energy. This energy is designated as $Q(\sigma_i)$ and is termed the activation energy of the process. From rate theory, the chance that a dislocation does receive enough energy to surmount the barrier is given by

$$\alpha(\sigma_i) = f \exp\left[\frac{-Q(\sigma_i)}{kT}\right] \tag{14}$$

where f is a frequency of the order of 10^8 to 10^{10} per second.

Therefore, after a time, t, the number of dislocations between σ_i and $\sigma_i + d\sigma_i$ which have not moved is given by

$$N(\sigma_i)\ d\sigma_i\ \exp\left[-\alpha(\sigma_i)t\right] \tag{15}$$

By utilizing equation (14), the number of these dislocations able to move and contributing to the creep rate is obtained by multiplying by αv, where v is a measure of the strain produced by the movement of the dislocation. The creep rate is then given by

$$\dot{\epsilon} = v\int_{\sigma}^{\infty} N(\sigma_i)\ \exp\left[-\alpha(\sigma_i)t\right]\alpha(\sigma_i)\ d\sigma_i \tag{16}$$

Mott and Nabarro have evaluated this integral in the following form;

$$\dot{\epsilon} = \text{Constant } T^{2/3}\left[\log_e(1 + ft)\right]^{2/3} \tag{17}$$

Since f is so large, it is evident that a much greater extension will take place from $t = 0$ to $t = 1$ second than for any time period after 1 second.

Pure exhaustion creep must, therefore, show an initial deformation greater than the total time-dependent extension. It would be expected that this type of creep is predominant at low stresses and relatively high temperatures where strain-hardening is not caused by generation of dislocations. Precipitation-hardened metals might be expected to

conform to this pattern. However, experimental evidence does not bear
out this expectation in some cases.

Generation creep.- In the previous section creep at low stresses
and low temperatures was discussed, and it was deduced that, eventually,
creep would cease because of the exhaustion of available dislocations.
It is probable that if the stress were high enough new dislocations could
be generated and the rate of creep would decrease because of physical
hardening. This type of transient creep has been investigated by Andrade
(references 4 and 13), who arrived at the relation

$$l = l_0\left(1 + \beta t^{1/3}\right) \tag{18}$$

or

$$\epsilon = \beta t^{1/3}$$

where

$$\epsilon = \frac{l - l_0}{l_0}$$

Although Andrade's work was entirely with polycrystalline material, the
same general shape of the curve has been found for single crystals.
Recently, Cottrell and Aytekin (reference 50) found for single crystals
of zinc that the $t^{1/3}$ law held very closely. However, they suggested
that, since the resolved shear stress remained constant by decreasing
the applied load, the Andrade equation be modified to include the instan-
taneous shear upon loading. The relation then becomes, in terms of shear
strain,

$$\gamma_t = \gamma_0 + \beta t^{1/3} \tag{19}$$

where γ_t is total shear strain at time t and γ_0 is instantaneous
shear strain upon loading.

Mott (reference 143) considers Andrade's relation (equation (18))
to fit some materials over a certain range of stress. In tests at rela-
tively large stresses, creep curves are obtained in which the instantaneous
deformation is less than the time-dependent deformation. This is associated
with the generation of dislocations as a result of strain-hardening. Under
these conditions, the change in the number of available dislocations is
given by

$$N_{available} = -N_{exhaustion} + N_{generation} \tag{20}$$

This particular range has not been adequately investigated, and as a result analytical relations have not been formulated.

Orowan (reference 155) introduced a new concept of transient creep using as a basis Becker's theory of flow (equation (12)). Because of the excellent correlation between experimental and calculated transient-creep curves a detailed description is given of Orowan's hypothesis. Although Orowan's relations have been applied to polycrystalline material, from the fact that the only creep he considers is transcrystalline and the effect of grain boundaries is disregarded, it is appropriate that his relations be treated in this section on single crystals.

The plastic stress-strain curve for the times considered (which are relatively short, < 60 min) takes the form shown in figure 10. The application of a stress OA produces an instantaneous strain AP. When this initial extension has stopped at P thermal stress fluctuations produce slight increases in stress which allow creep to proceed beyond P to the point Q. Now the yield strength of the metal due to work-hardening is BR so that the activation stress necessary is QR = ΔT. The Becker activation energy then becomes

$$\frac{V(BR - BQ)^2}{2G} = \frac{V(\Delta T)^2}{2G} \tag{21}$$

This assumes that strain-hardening during creep is the same as during rapid straining (which it is not). Probably the rise in yield stress is less and follows the curve PS. If this is so, then creep ceases to be a function of stress, strain, and temperature and becomes dependent upon the stress-strain history of the material.

It is evident that at the beginning of creep an activation is of longer range because of the many spots with similar activation energies. However, as the process continues and the gap between the yield and applied stresses widens, the strain produced will not be of the same magnitude. Therefore, the contribution of a thermal stress fluctuation is inversely proportional to the creep strain. The creep rate from the modified Becker-Orowan theory becomes

$$\dot{\gamma} = \frac{C}{(\Delta T)^2} \exp\left[-(\Delta T)^2 V/2GkT\right] \tag{22}$$

where C is a constant.

It is advantageous to use the slope of the stress-strain curve shown in figure 10, which is given by

$$\frac{d\tau}{d\gamma} = n = \tan \alpha \tag{23}$$

where n is often referred to as the strain-hardening exponent when obtained from the plot of true stress against true strain. The activation stress $\Delta\tau$ from figure 10 is equal to $n\gamma$. Equation (19) then becomes

$$\dot{\gamma} = \frac{C}{n^2\gamma^2} \exp\left(-n^2\gamma^2 V/2GkT\right) \tag{24}$$

Equation (24) can be integrated to give the relation between strain and time, thus

$$\frac{2VC}{2GkT} t = \gamma \exp\left(Bh^2\gamma^2/T\right) - \int_0^\gamma \exp\left(Bh^2\gamma^2/T\right) d\gamma \tag{25}$$

For short-time tests, the exponential in equation (24) becomes approximately constant (low values of γ) and the curve is then a cubic parabola. The relation then reduces to

$$\dot{\gamma} = \frac{d\gamma}{dt}$$

$$= \frac{C}{n^2\gamma^2}$$

or

$$\gamma = \frac{C}{n^2} t^{1/3} \tag{26}$$

Thus, for special cases, Orowan's expression for transient creep reduces to Andrade's empirical relation (equation (18)). Andrade (reference 13), discussing Orowan's analysis, stated that creep of single crystals of cadmium did not follow equation (24), in that a final length was not being asymptotically approached as expected from Orowan's theory. The creep of cadmium crystals followed the relation

$$\frac{da}{dt} = Ae^{c/a} \tag{27}$$

where A and c are constants, and a is the glide (relative displacement of planes unit distance apart). Andrade attributes failure of the Orowan theory to the fact that crystal recovery is not considered part of creep. The recovery theory states that the yield stress falls during a time dt by an amount proportional to dt. Thus,

$$d\sigma = r\ dt \tag{28}$$

where r is the rate of softening. If a constant stress is applied to a specimen, the material will harden at the same time that it softens. The creep rate is determined by the balance or unbalance between the two and is expressed by

$$\frac{d\epsilon}{dt} = \frac{r}{n} \tag{29}$$

Thus, if the stress is reduced, the creep rate should stop; however, Orowan (reference 155) cites experiments by Los where the creep rate has merely been reduced and there is no trace of any stoppage. It is believed, however, that the question whether creep stops is based on the magnitude of the reduction in stress. Experiments by Carreker, Leschen, and Lubahn (reference 39a), using greater increments of stress, showed a cessation of creep immediately upon removal of load. This lasted for large time intervals (10 min) and then creep resumed. This induction period can be explained on the basis of nucleation of slip bands by dislocations.

It is seen that the understanding of creep of single crystals has been only slightly enhanced by the aforementioned theories. The various theories are developed to fit only special cases and cannot be considered to be adequate. The tests on which the theories are based have been too few in number and cover only a limited range. It is believed that before adequate relations can be formulated more complete data are needed. If a single crystal can behave in a viscous manner, none of the present theories can be considered to hold without some modification. The thermal fluctuation theory of Orowan's would indicate that creep curves could never be duplicated because of the randomness of the process. The value of Mott's theory is lessened because of the assumptions involved in evaluating his integral.

TRANSITION FROM SINGLE TO POLYCRYSTALLINE METALS

Single crystals supposedly creep mainly by a slip mechanism. The question then arises of how the presence of grain boundaries affects creep. A grain boundary exists as a result of a difference in orientation of two adjacent grains. Thus, a grain boundary is a transition zone consisting of atoms acted upon by forces from the adjacent grains. The atoms in this zone might be at greater distances and thus possess lower strength. However, their apparent strength might be greater because of the lack of planes for slip to occur. Thus, grain boundaries do not increase the elastic modulus of a material, but only alter the range over which it is elastic.

It is expected that, because of the higher energy associated with grain boundaries, they will melt first. Hence, any property depending on melting point will be affected. Consequently, a specimen containing grain boundaries might appear stronger at low temperatures but, as the temperature is increased, the strength will decrease below that of a single crystal.

Chalmers (reference 43), investigating the effect of a boundary between two crystals, found that the yield point varied regularly with the angle between the two crystals. The strength was a minimum when the two lattices were similarly oriented and a maximum when the two lattices were at right angles. He then concluded that the boundary has no inherent strength and that the increase in strength can be best explained on the translational lattice theory.

In support of this conclusion, Cox and Sopwith (reference 51) calculated the strength of a randomly oriented polycrystalline material from single-crystal data and found excellent correlation with experimental results. Their results indicated that, although the shear strength of a single crystal is half of its tensile strength, the ratio of shear to tensile strength is increased to 0.577 for polycrystalline materials. Although there is some reason to doubt the similarity of the active slip planes in polycrystalline and single crystals, the results have been shown to be numerically correct. The increase in yield strength due to grain boundaries can be explained on the basis of blocking of slip of the active planes.

Betty (reference 27), noting the similarity between creep curves of single-crystal and polycrystalline lead, concluded that creep takes place within the grains. The curve for the polycrystalline lead was smoother because of the averaging effect of the grain boundaries. He noted that creep might be expected to be less in single crystals than in polycrystalline lead. This conclusion should be accepted only in the light that lead at room temperature is close to its melting point and the grain

boundaries should not be expected to improve the resistance to plastic flow.

Miller (reference 138) in tests on zinc concluded that the influence of the grain boundary is confined to the glide planes it intersects. It is conceivable that the grain boundaries might alter the stress distribution so that the shearing stress may never reach the critical value.

Hirst (reference 100) in a study of lead consisting of five crystals noted that deformation took place in each crystal as if it were a single crystal. The effect of the size of the grains affects the results quite markedly. In large-grained specimens, the deformation took place more rapidly at the boundaries. The mechanism of recrystallization also affected his results. This will be considered in greater detail later in this report.

It might then be expected that, under suitable conditions such as low temperatures, high-stress grain boundaries can improve creep resistance, while at high temperatures the grain boundary will weaken the metal. The calculation of the creep properties of polycrystals from single-crystal data is improbable, although approximations can be made. The effect of orientation of the grains is a factor only lightly investigated and merits further work.

CREEP OF POLYCRYSTALLINE METALS

The fact that nearly all metals are used in the polycrystalline condition has guided creep investigation in the majority of cases. Much of the earlier work centered about the secondary stage of the creep process, and many empirical formulas evolved because of the linear relationship between secondary creep rate and stress. The creep of polycrystals is extremely complicated and many factors are operative at once, which makes interpretation of results difficult. The influence of these factors will first be discussed so that proper weight may be given to the various items when considering the mathematical relations.

Metallographic Investigation

Most investigators agree that the primary stage of creep is marked by the formation of slip lines and rotation of crystals or crystallites. As time increases and the crystals harden, creep can begin in the grain or crystallite boundaries if the temperature is high enough. If it is not, then creep essentially ceases. Hanson (reference 96) describes the flow at high temperatures as "slipless" flow. He states that the crystals

appear to be swimming in their boundaries. The third or final stage is marked by reappearance of slip lines and cell rotation. Depending upon the temperature and strain rate, the fracture may be either transcrystalline or intercrystalline. It is thus apparent that flow within the grains by one or more mechanisms and in the grain boundaries may be taking place simultaneously depending on time, temperature, stress, and strain rate.

Grussard (reference 57) describes the creep of aluminum at various temperatures. At low temperatures (< 150° C), the slip bands form and grow to strike the grain boundaries. From temperatures of 150° C and up, in the case of very slow rates of creep, grain-boundary movement begins. At 200° C for rapid straining, slip bands are produced, while for lower rates of straining, grain-boundary movement is produced. The rate of straining has a similar effect on the mode of fracture. He also noted that metals having a hexagonal lattice have relatively longer primary-creep periods than cubic metals, probably because of the lesser number of available slip planes.

Greenwood (reference 91) found that creep is partly viscous and partly crystalline in his studies on lead. The proportional limit can be taken as an approximate guide to the critical stress above which the crystalline component becomes effective. At stress above the proportional limit, the creep curve is influenced by strain-hardening caused by generation and sticking of slip lines.

Wilms and Wood (reference 194) examined the nature of creep by X-ray and metallographic methods. They concluded that at room temperature the elements of flow are very small and deformation results from the relative transition of the elements between specific planes (slip planes). At higher temperature and slower strain rates, the grains break up into coarser elements which move without reference to specific planes. These elements have been termed cells and are a subgrain effect. These cells have similar orientation differing only by a matter of a few degrees. During creep, the cells appear to remain structurally perfect. The size of the cells is governed by the temperature and strain rate. It is to be expected that the larger the cell the smaller the effect of strain-hardening. Thus, the resistance to deformation at high temperatures will be determined by the relative proportion of cell and slip-band formation.

Effects of Metallurgical Variables on Creep

The effects of solid-solution-forming elements on the creep properties of a metal are not clear. Phillips (reference 158) found that, in lead, elements that form solid solutions decrease the creep resistance

in proportion to their solubility; conversely, it has been found that some alloying elements forming solid solutions with copper raise its creep resistance (reference 162). This is true provided the test temperature and time are below the softening or recrystallization range of the alloy. At higher temperatures the effects of alloying are negligible. These opposite findings appear to indicate that not enough data are available to allow general statements.

It is generally agreed that large-grained material is more creep resistant than fine-grained material. McKeown (reference 130) found in his study of lead that the increase of resistance to creep with increase of grain size becomes less marked as the mean grain area exceeds 1/2 square millimeter. Parker and Riisness (reference 157) found no effect of grain size in creep of copper at 200° C. However, their rates of strain were too fast and, therefore, the effect would not be expected to be so pronounced.

Clark and White (reference 45) found that for brass below a certain temperature range the fine-grained material was better; above it, the coarse-grained material was better. Hirst (reference 100) found that in large-grained lead specimens the deformation proceeds more rapidly at the grain boundary. It would be expected that recrystallization would begin in this region and increase the creep rate.

Recrystallization probably is the most important factor affecting the rate of creep of metals. At temperatures below the recrystallization range the metal is able to withstand considerable stress without continuous creep. In other words, the creep will tend to reach a limit. Within and above the recrystallization range almost any stress will cause continuous flow. If the specimen does not recrystallize immediately at the test temperatures, it is most probable that the third stage of creep will be initiated when recrystallization transforms a major portion of the specimen.

Hirst (reference 99) noticed that recrystallization of a single-crystal lead creep specimen begins at the edge and extends into the body of the specimen along the slip planes. At some stage, other crystals appear at the boundary of the first and the parent crystal. He states that crystals formed directly from the same parent grain have practically the same orientation with respect to the specimen axis.

Greenwood (reference 92) found that lead at room temperature will recrystallize under stress when it reaches 4- to 5-percent elongation at strain rates of 10^{-4} to 10^{-5} inch per inch a day. The creep rate then shows a marked increase. The recrystallization was observed to start at one or more centers and spread through the metal. Its progress could be noted since originally the slip lines stopped at the grain boundaries, but after recrystallization the slip line extended across

many of the original boundaries. He found that recrystallization was facilitated by vibration. With the same creep rate of 10^{-4} inch per inch per day, recrystallization will begin at 2.5 percent when a 50-cycle vibration is superposed. As one of the results of this investigation, it was concluded that neither recrystallization nor increase in stress is necessary for initiation of the third creep stage.

Effect of Stress and Temperature Cycling

Brophy (reference 33) found that cycling between 1800° F and lower temperatures increased the creep rate 70 times over that obtained by maintaining the temperature of 1800° F. He found that the magnitude of the increase in strain decreases as the maximum temperature and the temperature range of the cycle decreases. This effect is thought to be caused by a stress generated within the test bar as a result of a transverse thermal gradient produced during the relatively rapid cooling.

Fellows (reference 70) reports that varying the temperature $\pm10^\circ$ F every 7 minutes produced a creep rate six times as fast as a steady-temperature test at 1800° F. Also, the creep rate was much faster than that of a test run at 1810° F, the top temperature of the cycle. On this basis, he concluded that the maximum temperature cycle should be $\pm0.5^\circ$ F and the maximum deviation about the control point $\pm1.5^\circ$ F.

Relatively little work has been done toward investigating effect of stress cycling, although Foley (reference 74) states that cyclic loading increases creep deformation in the primary stage. It would be expected from purely theoretical reasoning that anything which upsets the balance of a metastable system would increase the rate of change.

Initial Stage of Creep

The initial stage of creep, which is made up of the elastic or instantaneous deflection and the declining creep-rate portion of the curve, has not been fully investigated. Probably the most complete empirical analysis of this range has been made by Andrade (references 3, 4, and 13) using constant-stress tests. The expression which seemed to fit the curves best was

$$l = l_0\left(1 + \beta t^{1/3}\right) \exp k'T \tag{30}$$

where β and k' are constants.

The term $\beta t^{1/3}$ represents the transient flow, while the term containing k' describes the permanent or viscous flow. It has been

found that at low temperatures k' is equal to zero, at intermediate temperatures the two flows are of similar importance, and at high temperatures the k' flow predominates. At constant temperature, $k' = a \exp b\sigma$ where a and b are constants. If the metal has been strained sufficiently before testing, it may be in the condition for k' flow almost immediately. The β and k' stages are not considered to be independent phenomena, although this has not been definitely proven. For longer tests (> 40 hr), the transient flow falls below that given by $t^{1/3}$ and, possibly, for long times should be replaced by a constant.

Tapsell and Prosser (reference 181) plotted creep data using a linear strain ordinate and a logarithmic time ordinate. The equation for the resulting straight lines is given as

$$\epsilon = a \log (1 + bt) \tag{31}$$

where a and b are constants.

Chevenard (reference 44) represented both the first and second stages of creep in a single expression. This is done by adding to Tapsell's relation (equation (31)) a term dependent upon the constant rate. The relation then becomes

$$\epsilon = a \log (1 + bt) + V_0 t \tag{32}$$

where V_0 is the minimum creep rate.

Kanter (reference 108) noticed that the initial creep stage for steel between the temperatures of 750° and 1000° F is linear when plotted on logarithmic coordinates. The relation between creep, strain, and time is then

$$\epsilon = at^n \tag{33}$$

where n can be termed the creep index of strain-hardening.

These early curve-fitting relations varied quite widely with materials, temperature, and stress. It was Andrade who first noticed during tests on lead that most creep curves could be duplicated by adjusting the stress and temperature. Probably the most satisfactory quantitative mechanisms of primary or transient creep have been advanced by Mott and Orowan. These have been discussed under single crystal creep and include Andrade's relation as a special case.

Bailey (reference 20) considered that when the initial stress is applied it does not distribute itself uniformly throughout the metal and

the high creep rate occurs during the time the metal is adjusting itself to the load. Owing to differences in orientations, various grains yield at different stresses; consequently, they deform and strain-harden at different rates. Those particular grains which are the most highly stressed tend to creep more readily than others. However, these also strain-harden more rapidly and tend to resist further creep. As these grains become more numerous, the stress tends to become uniform and the curve of strain against time tends to fall off. There is probably another process taking place during this stage which is relatively minor until larger strains and times are reached. This competing mechanism is known as recovery or softening. Softening becomes greater in importance during the second stage of creep when, according to some investigators, the steady rate is a balance between softening and strain-hardening.

Bailey's theory does not explain why single crystals and polycrystals have similar curves in the initial stage.

Second Stage of Creep

The coincidence that the creep rate sometimes becomes constant after an initial transient period has been widely used as an evaluating tool for creep-resistant alloys. Many theories attempting to describe creep phenomena have only considered this stage. No attempt to review all the theories will be made, but only those theories which have been widely used or which attempt to explain creep in fundamental terms will be treated.

McVetty (references 133 to 135) noted that families of creep curves at constant temperature and varying stress indicated that a creep rate in excess of a certain minimum rate is decreased nearly geometrically as the time increased arithmetically. The relation between creep rate and time is then

$$\dot{\epsilon} - \dot{\epsilon}_0 = c \exp(-\alpha t) \tag{34}$$

where $\dot{\epsilon}_0$, c, and α are constants.

If equation (34) is integrated with respect to time, the relation between strain and time becomes

$$\epsilon = \epsilon_0 + \dot{\epsilon}_0 t - \frac{c}{\alpha} \exp(-\alpha t) \tag{35}$$

The asymptote to this curve as t becomes large is

$$\epsilon = \epsilon_0 + \dot{\epsilon}_0 t \tag{36}$$

The constant ϵ_0 is the intercept on the strain axis and $\dot{\epsilon}_0$ the slope. The assumption that an asymptote exists is equivalent to stating that second-stage creep is viscous in nature.

A more exact relation later adopted by McVetty (reference 136) and originally proposed by Nadai (reference 145) is the hyperbolic sine function. This relation is found to hold for very low values of stress. It is given by

$$\dot{\epsilon} = \dot{\epsilon}_0 \sinh \frac{s}{s_0} \tag{37}$$

where s is the stress and $\dot{\epsilon}_0$ and s_0 are constants. Equation (37) can also be written as

$$\dot{\epsilon} = \frac{\dot{\epsilon}_0}{2}\left[\exp\left(\frac{s}{s_0}\right) - \exp\left(-\frac{s}{s_0}\right)\right] \tag{38}$$

which, for large values of s/s_0, becomes

$$\dot{\epsilon} = \frac{\dot{\epsilon}_0}{2} \exp \frac{s}{s_0} \tag{39}$$

which is the common logarithmic rate law.

Bailey (reference 20), Tapsell (reference 181), and others have used the expression

$$s = B\left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0}\right)^n \tag{40}$$

However, the objection to this is that it does not pass through the origin. Extrapolation toward lower rates using logarithmic plots is equivalent to assigning the initial stress a for smaller creep rate than would be expected.

It is to be emphasized that all these tests which fit these equations are run at constant load and $\dot{\epsilon}$ represents the secondary creep rate. These relations are the result of graph juggling and should not be used for extrapolation purposes. They are equations of experimental curves and, although sometimes they are close approximations, under different conditions they may differ by several orders of magnitude.

Kanter (reference 109) presented a theory embodying the principles of the Dushman-Langmuir theory of solid diffusion. The grains are assumed to swim in their own boundaries and the strain takes place by self-diffusion of the grain-boundary atoms. The Dushman-Langmuir equation states that

$$D = \frac{jQ}{N_o h} \, d^2 \, \exp \, (-Q/RT) \tag{41}$$

where

D   diffusion coefficient

j   mechanical equivalent of heat; 4.185×10^7 ergs per calorie

Q   activation energy

N_o Avogadro's number; 6.061×10^{23}

h   Planck's constant

d   interatomic distance

Using equation (41) as a basis and considering that creep is taking place within the recrystallization range, Kanter arrives at the following expression:

$$\dot{\epsilon} = \frac{j(Q - T\Delta S)}{N_o h} \left(\frac{1}{2} \sqrt{2} \, d\lambda\right) \frac{\sigma}{\sigma_o} \, \exp\left(-\frac{Q - T\Delta S}{RT}\right) \tag{42}$$

where

λ number of lattice imperfections traversed per unit length

σ_o cohesive molecular force

Values of the activation energy Q for creep were found to be of the same order as those for recrystallization and self-diffusion. This expression, although appearing to fit the data well, has not gained wide popularity. The comparing of creep with self-diffusion, although they are fundamentally the same since both involve atomic movements is rather difficult to visualize. The grains are eliminated from consideration and even in the recrystallization range some crystalline flow must be expected.

Kauzmann (reference 112), utilizing Eyring's theory of shear rates of liquids, restated the equation to describe the creep of metals. The creep is postulated to take place by the movement of units of flow. The sizes of these units are chiefly fixed by the lattice dimensions. A model of the mechanism of flow by shear is shown in figure 11.

The motion of the units of flow through the stages a, b, and c results in a shear strain λ/L. The resulting rate of flow is given by

$$S = \frac{\lambda}{L} V \tag{43}$$

where λ is the average distance in the shear direction moved in one jump, L is the average distance between layers of flow, and V is the net number of jumps in the shear direction per second.

By assuming that an externally applied stress modifies the potential-energy curve in such a way as to lower the activation energy required for the units of flow to move, the following equation is derived:

$$\dot{\gamma} = \frac{\lambda}{L} \frac{kT}{h} e^{\Delta S/R} e^{-\Delta Q/RT} \left(e^{A l f/kT} - e^{-A l f/kT} \right) \tag{44}$$

where

A   projected area of unit of flow on shear plane

l distance through which shear stress acts in carrying unit of flow from normal to activated state

f   shear stress per unit area

If $A l f \ll kT$, then

$$e^{A l f/kT} - e^{-A l f/kT} = \frac{A l f}{kT} \tag{45}$$

and equation (44) becomes

$$\dot{\gamma} = \frac{2\lambda l A}{L h} e^{\Delta S/R} e^{-\Delta Q/kT} f \tag{46}$$

This is the equation for true viscous flow where the shear rate is proportional to the shear stress. However, if $A l f \gg kT$, then equation (44) becomes

$$\dot{\gamma} = \frac{\lambda kT}{Lh} e^{\Delta S/R} e^{-Q/RT} e^{A l f/kT} \tag{47}$$

In terms of the ordinary tensile creep test, equation (47) becomes

$$\dot{\epsilon} = \frac{2\lambda kT}{3Lh} e^{\frac{\Delta S}{R}} e^{-\frac{Q}{RT}} e^{\frac{qA/\sigma}{2kT}} \tag{48}$$

where

$$\dot{\epsilon} = \frac{2}{3} \dot{\gamma}$$

$$\sigma = 2\tau$$

Here $\tau = f/q$ where q is a stress-concentration factor which converts the microscopic stress f into the macroscopic applied stress. It is to be noted that upon reducing equation (46) to simple terms, it takes on the form

$$\log_e \dot{\epsilon} = A + B\sigma \tag{49}$$

where A and B are constants. Thus, any creep curve which is a straight line on semilogarithmic paper can be described by equation (46).

It is interesting to examine representative values of the various terms in equation (46). The energy of activation varies considerably with different metals ranging from 3800 calories per mole for lead to 102,000 calories per mole for cast steel. It is roughly of the same order as the activation energy for self-diffusion and recrystallization. The entropy of activation ΔS would be expected to be approximately zero, but it is found to be a large negative number in the neighborhood of 50 to 100 calories per degree per mole.

The value of qAl ranges from 10^3 to 10^5 cubic Angstrom units. The division into separate values of q, A, and l is ill-defined and one of the weak points of the theory.

The effects of grain size and grain boundaries have not been considered by Kauzmann, although he recognizes the importance of these

variables. The use of the mechanism postulating microscopic units of flow, although interesting, has not been supported by metallographic investigation, which shows the flow to be caused by macroscopic cells. (See reference 194.)

Davis (reference 62) found that the Kauzmann theory holds for creep rates of copper tested between 130° and 235° C. However, at lower temperatures (80° C) the curve deviates from the straight line (semilogarithmic coordinates). It was found that, for these cases, the logarithmic plot or hyperbolic-sine plot fits equally well. This is as expected, since the equation designed for shear of liquids is less able to describe flow of metals as they acquire more "personality." In other words, at high temperatures, the metal loses some of its distinguishing traits, such as elasticity, yield point, and definite planes of slip, and tends to resemble a liquid. However, at low temperatures these qualities become more pronounced and the metal acts less like a homogeneous substance.

Dushman, Dunbar, and Huthsteiner (reference 63), using Eyring's theory independently, arrived at an expression similar to that of Kauzmann, and only reviewed the latter's theory. Dushman's final equation is given by the relation

$$\dot{\epsilon} = \frac{4\lambda kT}{3Lh} e^{\Delta S/R} e^{-\Delta Q/RT} \sinh \frac{j\alpha\sigma}{2RT} \tag{50}$$

where α is the volume of unit of flow per gram atom. In terms of Kauzmann's unit of flow, $\alpha = 6.023 \times 10^{23}$ qΛl, and j is a constant equal to 2.39×10^{-8} calorie per erg. The rest of the terms have the same meaning as in equation (44). When $j\alpha\sigma/2RT \geq 1.6$, equation (50) can be written as

$$\dot{\epsilon} = \frac{4\lambda kT}{3Lh} e^{\Delta S/R} e^{-\Delta Q/RT} e^{j\alpha\sigma/2RT} \tag{51}$$

This states that the logarithm of the strain rate is a linear function of the stress. If $j\alpha\sigma/2RT \leq 0.50$, the strain rate is found to vary linearly with the stress.

A number of data on various metals are presented which appear to follow these relations very well. Metallographic analysis of the test specimens indicated that, in the case of aluminum, the elongation was accompanied by the formation of slip planes. The distance between planes

increases as the temperature increases. Fracture was also observed to occur along these planes.

Betty (reference 28) found that in tests on Inconel and Monel at 800° to 1000° F the unit of flow increased with temperature. The data followed equation (51) very closely.

MacGregor and Fisher (reference 125), using Eyring's rate theory as applied by Kauzmann and Dushman to secondary creep rates, find that the following expression is consistent with data:

$$\frac{\dot{\delta}}{\dot{\delta}_o} = Ce^{-f(\sigma)/T} \tag{52}$$

where

$\dot{\delta}$ true strain rate

σ true stress

$\dot{\delta}_o, C$ constants

The stress function is then defined as a function of the velocity and modified temperature

$$\sigma = f(T_m) = f\left[T\left(1 - k \log_e \frac{\dot{\delta}}{\dot{\delta}_o}\right)\right] \tag{53}$$

where k is a constant. Equation (50) states that a stress corresponding to a test at a strain rate $\dot{\delta}$ and temperature T is the same as for a test at $\dot{\delta}_o$ and a temperature $T\left(1 - k \log_e \frac{\dot{\delta}}{\dot{\delta}_o}\right)$.

Although this formulation is based on Eyring's rate theory, it has no fundamental foundation other than that it fits the experimental curves in a limited number of cases. Its usefulness is lessened by the number of constants in the formula. It is believed its greatest applicability is to short-time constant true strain-rate tension tests.

Nowick and Machlin (reference 152) have formulated a theory for secondary creep based on the dislocation theory. They consider the dislocations to move and pass out of the lattice; therefore, new dislocations must be generated at points of stress concentration existing in the crystal. As the dislocations move through the crystal, they increase in length and move in the slip direction on the slip plane until they become stuck.

In setting up the equation, it is assumed that all slip takes place within the crystal. In applying chemical-rate theory, it is also assumed the reactant is the perfect lattice at the region of high stress concentration. The activated complex is the unstable array of atoms and the product is the dislocation.

Considering the creep rate to be a function of the rate of generation of dislocations, this rate is given by

$$\dot{\epsilon} = \left(d_1/L\right)Rg \tag{54}$$

where

$\dot{\epsilon}$ tensile creep rate

d_1 lattice spacing in slip direction

L spacing between imperfections

Rg rate of generation for a single source

The rate of generation Rg for a positive dislocation is given by

$$Rg^+ = \frac{kT}{h} P^+ \tag{55}$$

and for a negative dislocation

$$Rg^- = \frac{kT}{h} P^- \tag{56}$$

where P^+ and P^- are the probabilities of formation of positive and negative activated complexes. The elastic energy involved in thermal oscillation is given by

$$W = V\tau_o^2/G \tag{57}$$

where

W elastic energy

V atomic volume

G shear modulus

τ_o shear stress resulting from imperfections

In order for a dislocation to be generated, thermal oscillation and external stress must produce a certain amount of energy. The amount of thermal energy necessary is given by

$$W_t = (V/G)(\tau_o - \tau_s)^2 \qquad (58)$$

where τ_s is the shear stress imposed on dislocation.

To obtain the shear stress in terms of displacement, Hooke's law is used, resulting in

$$\tau_o = Gx\delta/d_1$$

$$= Gxf' \qquad (59)$$

where x is the ratio of lattice spacing in slip direction to lattice spacing normal to slip direction d_1/d_2 and δ, the shear displacement of the atom, is equal to some fraction f of d_1. Expanding equation (58) and substituting equation (59) for τ_o result in

$$W_t = VGx^2f'^2 - 2Vxf'\tau_s + \frac{V}{G}\tau_s^2 \qquad (60)$$

Nowick and Machlin state that τ_s^2 is very small for rigid materials and may be neglected.

By making use of Taylor's theory of work-hardening, the term τ_s was evaluated. By considering that every dislocation has a stress field about it designated as τ_b and this is subtracted from the externally applied stress τ_e, then τ_s may be expressed by

$$\tau_s = q(\tau_e - \tau_b) \qquad (61)$$

where q is a stress-concentration factor at the generating source. By making further assumptions as to the probability of oscillation in the different directions, they arrive at the formula giving the rate of generation of dislocations,

$$R_g = \frac{2kT}{h} \exp\left[-(VGx^2f^2 + CT)/kT\right] \sinh\left[2qVxf'(\tau_e - \tau_b)/kT\right] \qquad (62)$$

where C is a constant, approximately 40×10^{-16} erg per °K for common metals. By substitution into equation (54), the secondary rate of creep becomes

$$\dot{\epsilon} = \frac{2d_1}{L} \frac{kT}{h} \exp\left[-\left(VGx^2f^2 + CT\right) kT\right] \sinh\left[qVxf'\left(\sigma - 2\tau_b\right)kT\right] \quad (63)$$

Notice that σ the applied stress has been substituted for $2\tau_e$ on the assumption that slip occurs in planes of maximum shear stress.

For high values of σ, the hyperbolic sine function approximates an exponential; consequently,

$$\dot{\epsilon} = \frac{2d_1 kT}{Lh} \exp\left[-\left(VGx^2f^2 + CT\right)\Big/kT\right] \exp\left[\frac{qVxf'}{kT}\left(\sigma - 2\tau_b\right)\right] \quad (64)$$

Reduced to a general type of relation, equation (64) becomes

$$\log_e \dot{\epsilon} = \log_e \frac{d_1 kT}{Lh} - \frac{A + BT}{kT} + \frac{D\sigma}{kT} \quad (65)$$

which states that logarithmic creep rate plotted against stress should be a straight line.

Data are presented in the literature to show the validity of the equation. The calculations show that a newly generated dislocation is but one atomic distance long. The analysis is extremely interesting and indicates much work needs to be done before even the second stage of creep can be explained.

Third Stage of Creep

The reason for the initiation of the third or rapid creep stage is the center of much disagreement. There have been two main reasons why a metal which is increasing in length steadily with time suddenly extends quite rapidly and fractures. The first, originally proposed by Andrade, is that necking of the specimen begins and the cross section is reduced rapidly, and, consequently, the stress increases rapidly. The inflection point would correspond to the maximum-load point in a short-time tensile

test. The second, also recognized by Andrade, is structural instability. Recrystallization, precipitation, or some other phenomenon may occur and greatly facilitate the flow of the metal.

Sully (reference 180) found the third stage to be present in compression tests. Since the area increases and the stress decreases, the necking theory cannot be applied. He suggests that the third stage is initiated by atomic rearrangement in the most heavily strained zones adjacent to the crystal boundaries. The initiation is determined by the amount of strain undergone by the specimen. The point of initiation was found to be the same as in tension.

Flanigan, Tedsen, and Dorn (reference 73) found that the steepness of the curve in the third stage increases with increasing temperature. This could be explained using the recrystallization mechanism, for, increasing temperatures above a lower critical, the rate of recrystallization increases very rapidly.

Although there are numerous relations describing the initial and second stages of creep, little consideration has been given to the third stage. Recently it has been shown (reference 162) that the initiation of the third stage can be approximated by the use of a relation describing the rate of recrystallization of the specimen. Thus, when recrystallization has caused the yield strength of a previously cold-worked metal to drop to the test stress, conditions are favorable for initiation of the third stage.

CREEP UNDER COMBINED STRESSES

The framework of modern interpretation and utilization of combined-stress creep testing was laid by Bailey (reference 20) in 1935. He considered creep to be due to shear and to be uninfluenced by the stress acting perpendicularly on the plane of shear. Creep is not confined to planes of maximum shear but also occurs on planes of lower shear. Likewise, the creep on one plane is influenced by the shear stresses on other planes. In other words, it is influenced by the general state of stress

represented by the elastic shear-strain energy. On this basis, Bailey proposed the following relations:

$$\dot{\epsilon}_1 = \frac{A}{2}\left[\frac{1}{2}(\sigma_1 - \sigma_2)^2 + \frac{1}{2}(\sigma_2 - \sigma_3)^2 + \frac{1}{2}(\sigma_3 - \sigma_1)^2\right]^m\left[(\sigma_1 - \sigma_2)^{n-2m} - (\sigma_3 - \sigma_1)^{n-2m}\right]$$

$$\dot{\epsilon}_2 = \frac{A}{2}\left[\frac{1}{2}(\sigma_1 - \sigma_2)^2 + \frac{1}{2}(\sigma_2 - \sigma_3)^2 + \frac{1}{2}(\sigma_3 - \sigma_1)^2\right]^m\left[(\sigma_2 - \sigma_3)^{n-2m} - (\sigma_1 - \sigma_2)^{n-2m}\right] \qquad (66)$$

$$\dot{\epsilon}_3 = \frac{A}{2}\left[\frac{1}{2}(\sigma_1 - \sigma_2)^2 + \frac{1}{2}(\sigma_2 - \sigma_3)^2 + \frac{1}{2}(\sigma_3 - \sigma_1)^2\right]^m\left[(\sigma_3 - \sigma_1)^{n-2m} - (\sigma_2 - \sigma_3)^{n-2m}\right]$$

where $\dot{\epsilon}_1$, $\dot{\epsilon}_2$, and $\dot{\epsilon}_3$ are the tensile creep rates in the directions 1, 2, and 3 and A, m, and n are constants to be determined from tensile and torsion tests.

Soderberg (references 173 to 176) argued that Bailey's relation contained an unnecessary constant and derived a set based on Saint Venant's relations. His expressions for creep under compound stress are

$$\dot{\epsilon}_1 = \frac{A}{2\left(\frac{n+1}{2}\right)}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{\frac{n-1}{2}}\left[(\sigma_1 - \sigma_2) - (\sigma_3 - \sigma_1)\right]$$

$$\dot{\epsilon}_2 = \dots \dots \dots \dots \qquad (67)$$

$$\dot{\epsilon}_3 = \dots \dots \dots \dots$$

Bailey, however, contends that Soderberg's relations are but a special case where $n - 2m$ has a value of unity or $m = (n - 1)/2$.

Coffin, Shepler, and Chernisk (reference 49) use the maximum shear-stress theory to calculate primary creep in combined loads. They assume for steady-state creep the relation

$$\dot{\bar{\epsilon}} = A \exp (b\tau)\bar{\sigma}^n \tag{68}$$

where $\dot{\bar{\epsilon}}$ and $\bar{\sigma}$ are the effective strain rate and stress. Since, under the maximum shear law,

$$\dot{\epsilon}_1 - \dot{\epsilon}_3 = \dot{\bar{\epsilon}} \tag{69}$$

the general expression becomes

$$\dot{\epsilon}_1 = Be^{bT}\left(\sigma_1 - \sigma_3\right)^{n-1}\left[\sigma_1 - \frac{1}{2}(\sigma_2 + \sigma_3)\right] \tag{70}$$

Comparing Coffin's theory with Soderberg's distortion-energy theory, it is found that

$$\frac{\dot{\epsilon}_{\text{max shear}}}{\dot{\epsilon}_{\text{D.E.T.}}} = \left(\frac{\bar{\sigma}_{\text{max shear}}}{\bar{\sigma}_{\text{D.E.T.}}}\right)^{n-1} \tag{71}$$

The advantage claimed for the maximum shear theory is that it is easier to use.

Johnson (reference 107) studied creep in isotropic metals under various ratios of σ_1 and σ_2 from pure tension to pure torsion. He found that the creep rates appeared to follow the Soderberg theory. The creep strain followed the Huber-Von Mises-Hencky shear-strain energy criterion of yielding. Anisotropic metals have not been considered, although it is suspected that many of the metals considered isotropic because of their elastic properties are not so in their flow properties.

Siegfried (reference 166) discussed effects of notches on creep strength of brittle and ductile materials. The notched pieces were found to have greater strength for the same failure time as long as the material was ductile. Discrepancies were indicated in the picture and the need for more work was evident.

EFFECT OF ENVIRONMENT ON CREEP PROPERTIES

Andrade (reference 10) studied the effect of α-ray bombardment on
the creep of single-crystal wires of cadmium. He found the rate to
increase to several times the value obtained without bombardment. The
greater the prior strain, the smaller the increase. His explanation was
that glide is initiated by the α particles. Thus, α-ray bombardment is
a sensitive index as to whether glide is taking place by activation of
new planes or continued glide in active planes.

Nabarro (reference 144) discussed creep phenomena to be expected in
a metal when under neutron bombardment. This creep is a result of the
motion of interstitial ions and of the vacant lattice sites from which
they have been ejected. Mathematical relations are developed, although
no supporting data are presented. The discussion is speculative in
nature.

Andrade (reference 14) studied the effect of immersion in various
solutions on the creep rate of single-crystal wires of cadmium. He
found an increase of 20 times in the creep rate by immersion in a com-
mercial cadmium-plating solution. The effect was immediate, so that
diffusion was not concerned. Cadmium sulfate and chloride were found
to have similar effects. However, cadmium nitrate caused an immediate
increase followed by a cessation of flow. The wire was hardened to the
extent that almost double the stress was required to initiate flow again.
This hardening may be due to a surface film, probably oxide. Much needs
to be done in this field before quantitative explanations can be given.

Battelle Memorial Institute
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TABLE 1.- THEORIES AND CORRESPONDING EQUATIONS OF CREEP

Author of theory	Equation	Similar equation
Kauzmann (reference 112)	$\dot{\epsilon} = \frac{2}{3} \frac{\lambda}{L} \frac{kT}{h} e^{-\Delta F/RT} e^{qA\ell\sigma/2kT}$	Equation (8) where $$\frac{q\sigma}{2} = f$$
Dushman, Dunbar, and Huthsteiner (reference 63)	$\dot{\epsilon} = \frac{4}{3} \frac{\lambda}{L} \frac{kT}{h} e^{-\Delta F/RT} \sinh \frac{j\alpha\sigma}{2RT}$	Equation (6a) where $$2\sigma = f$$ $$\alpha = A\ell$$
MacGregor (reference 125)	$\dot{\delta} = \dot{\delta}_0 C e^{-f(\sigma)/T}$	Equation (8) where $$\dot{\delta}_0 = \frac{kT}{h}$$ C = Some constant $$f(\sigma) = \frac{fA\ell}{k}$$
Nowick and Machlin (reference 152)	$\dot{\epsilon} = \frac{2d_1}{L} \frac{kT}{h} \exp\left[-v\left(Gx^2f'^2 + CT\right)/kT\right] \times$ $\sinh\left[qVxf'(\sigma - 2\tau_b)/kT\right]$	Equation (6a) where $$\exp\left[-v\left(Gx^2f'^2 + CT\right)/kT\right] = e^{-\Delta F/RT}$$ $$Vxf' = A\ell$$ $$q(\sigma - 2\tau_b) = f$$
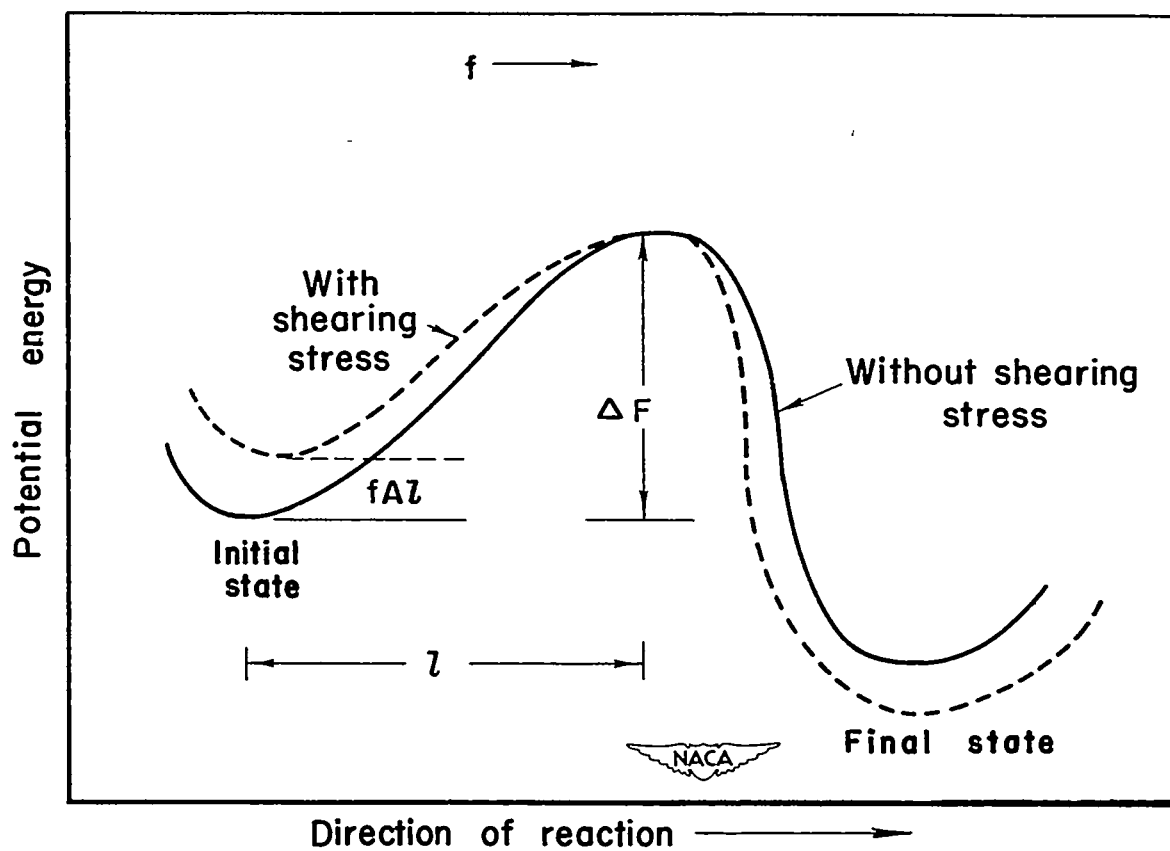
Figure 1.- Effect of stress on course of reaction.
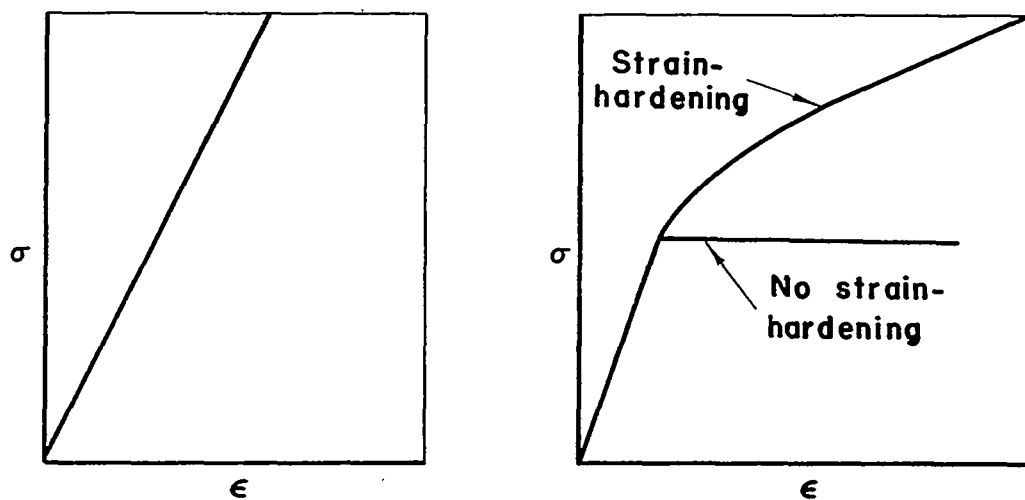
(a) Elastic material.

(b) Material with yield
point.
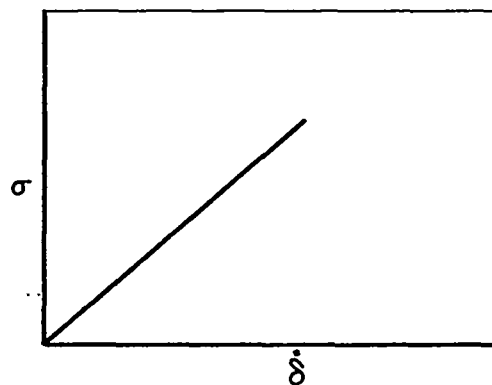
(c) Viscous material.

Figure 2.- Behavior of materials under stress.
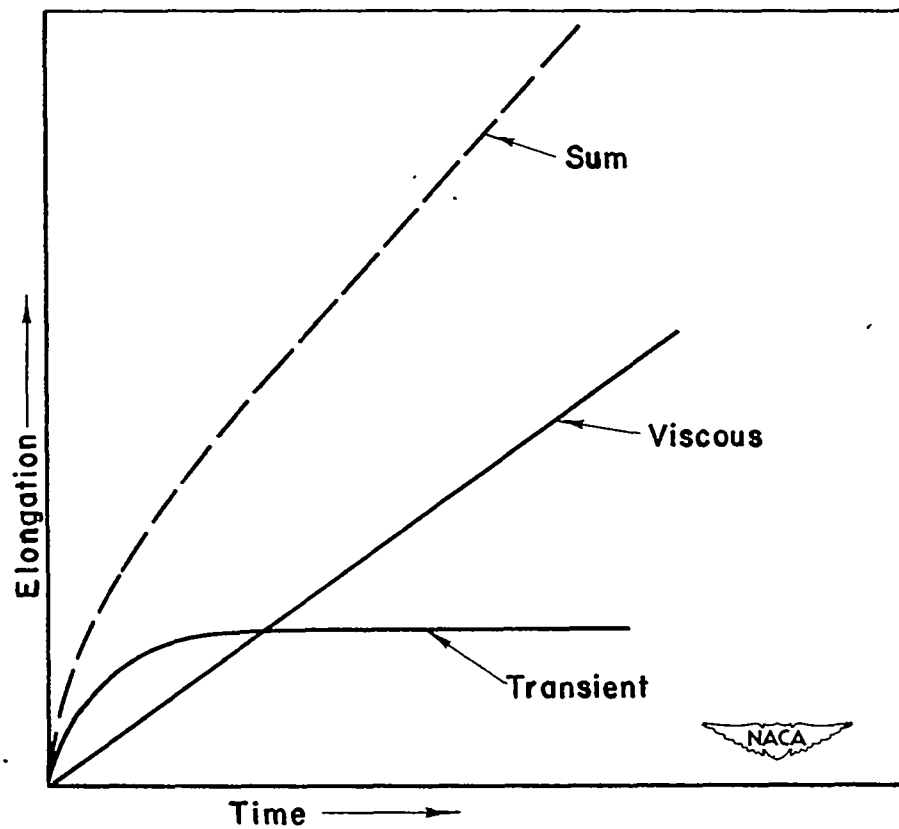
Figure 3.- Division of creep into transient and viscous components.
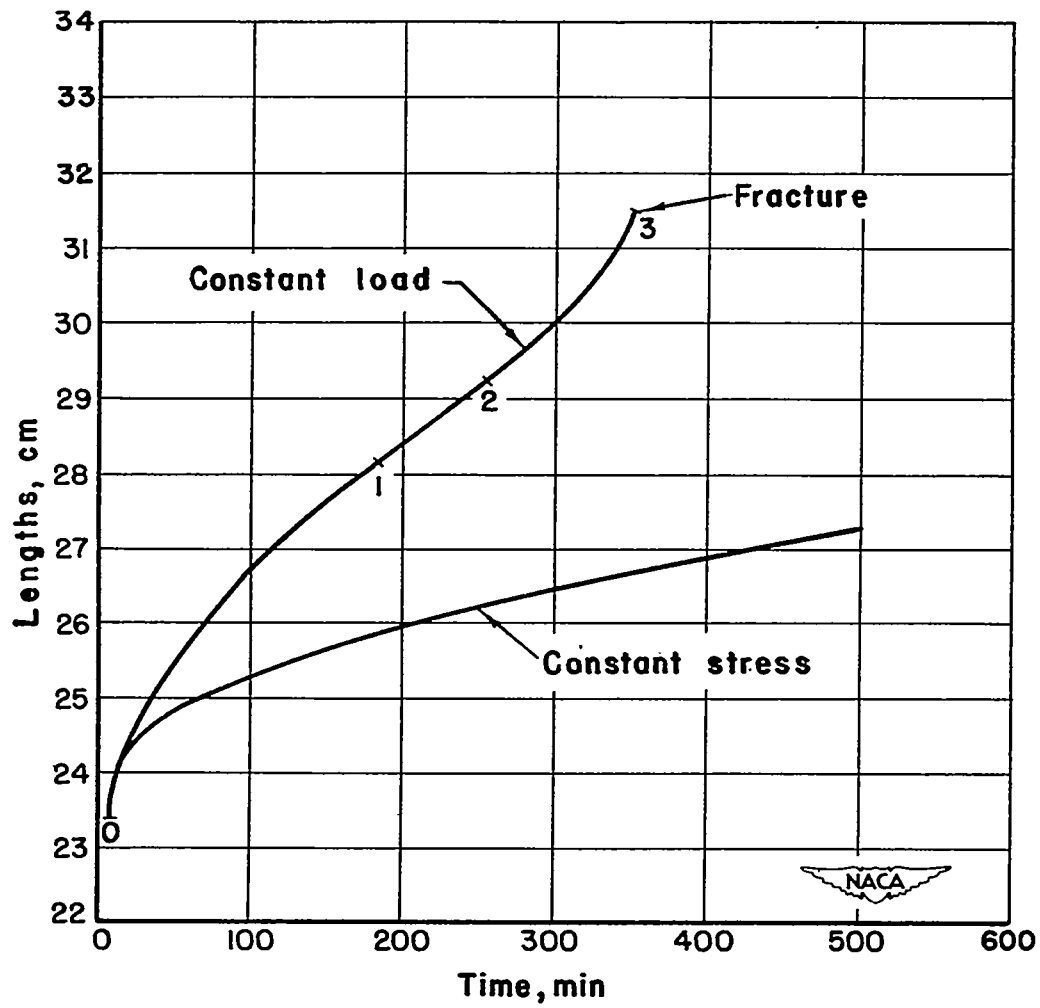
Figure 4.- Comparison of constant-load and constant-stress tests on
lead at 15° C. (From Andrade, reference 3.)
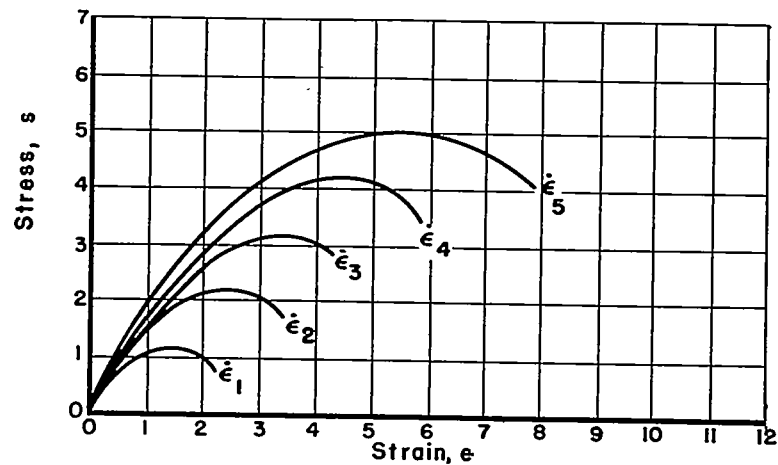
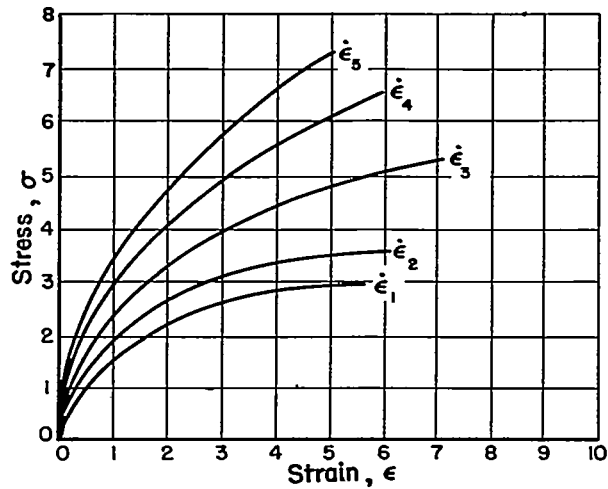(a) Curves of strain against stress for various constant strain rates.
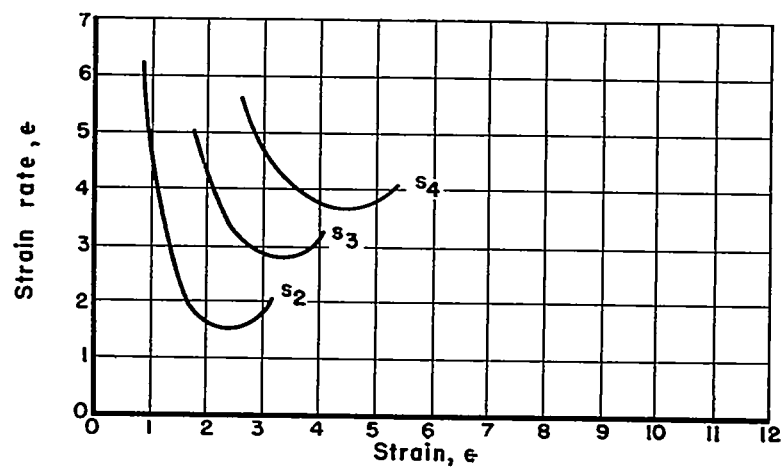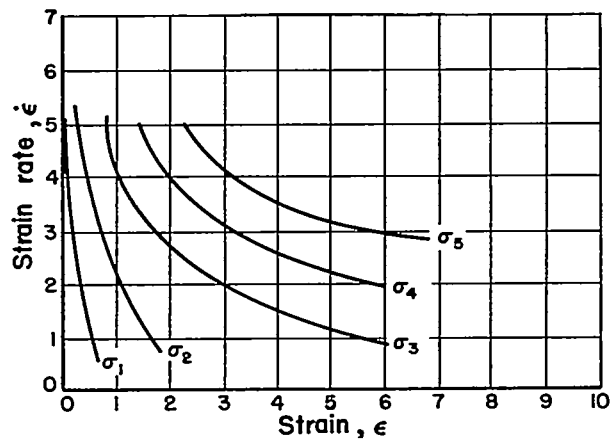
(b) Curves of strain against strain rate for various constant loads.
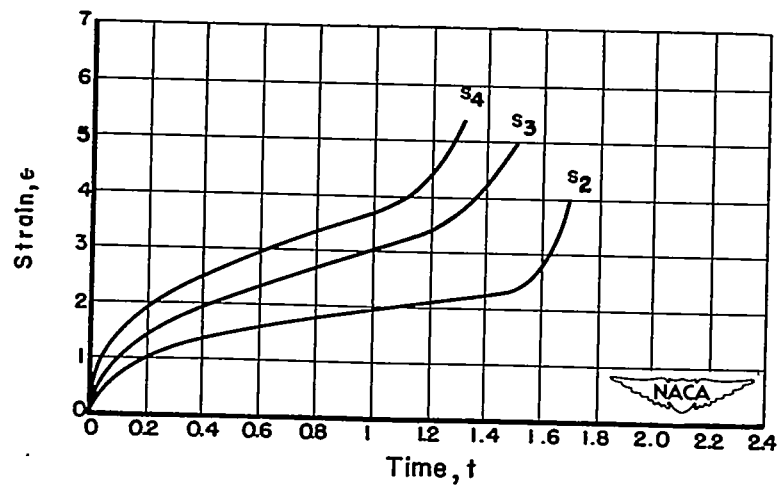
(c) Curves of strain against time for various constant loads.

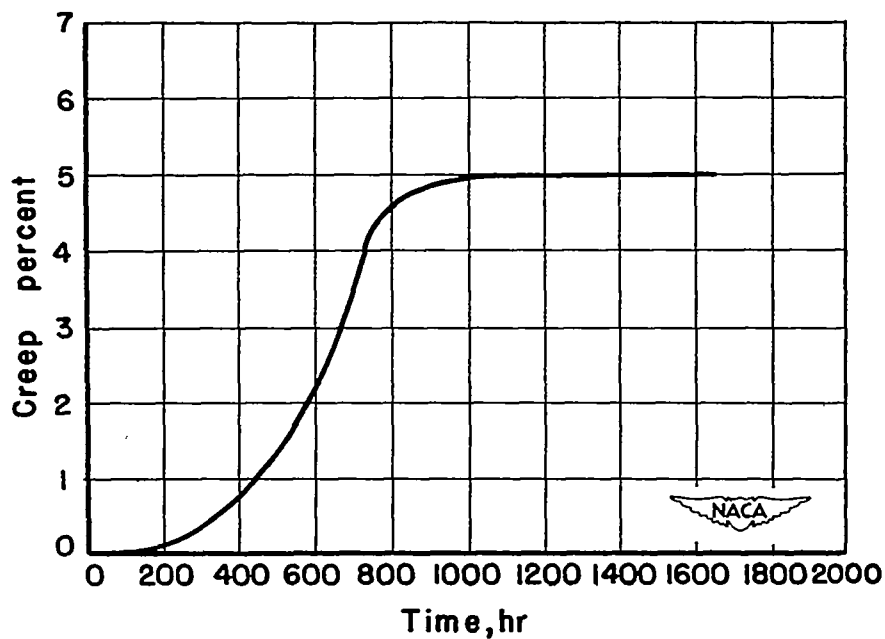Figure 5.- Creep curves derived from nominal stress-strain data.

(a) Curves of stress against strain for various constant strain rates.

(b) Curves of strain rate against strain for various constant stresses.
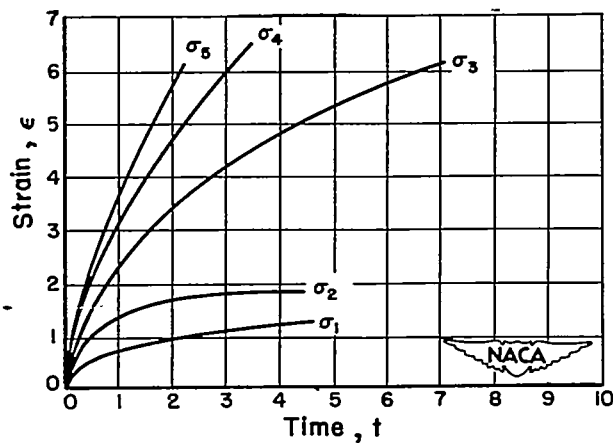
(c) Curves of strain against time for various constant stresses.

Figure 6.- Creep curves derived from true stress-strain data.

Figure 7.- Time-deformation curves for 70-30 brass at room temperature.
Maximum resolved shear stress, 1100 grams per square millimeter.
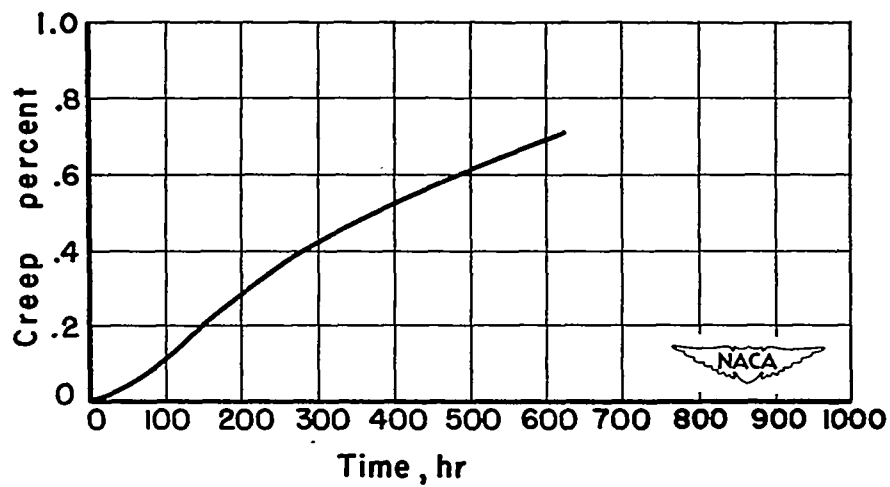

Figure 8.- Time-deformation curve for 70-30 brass at 700° F. Maximum
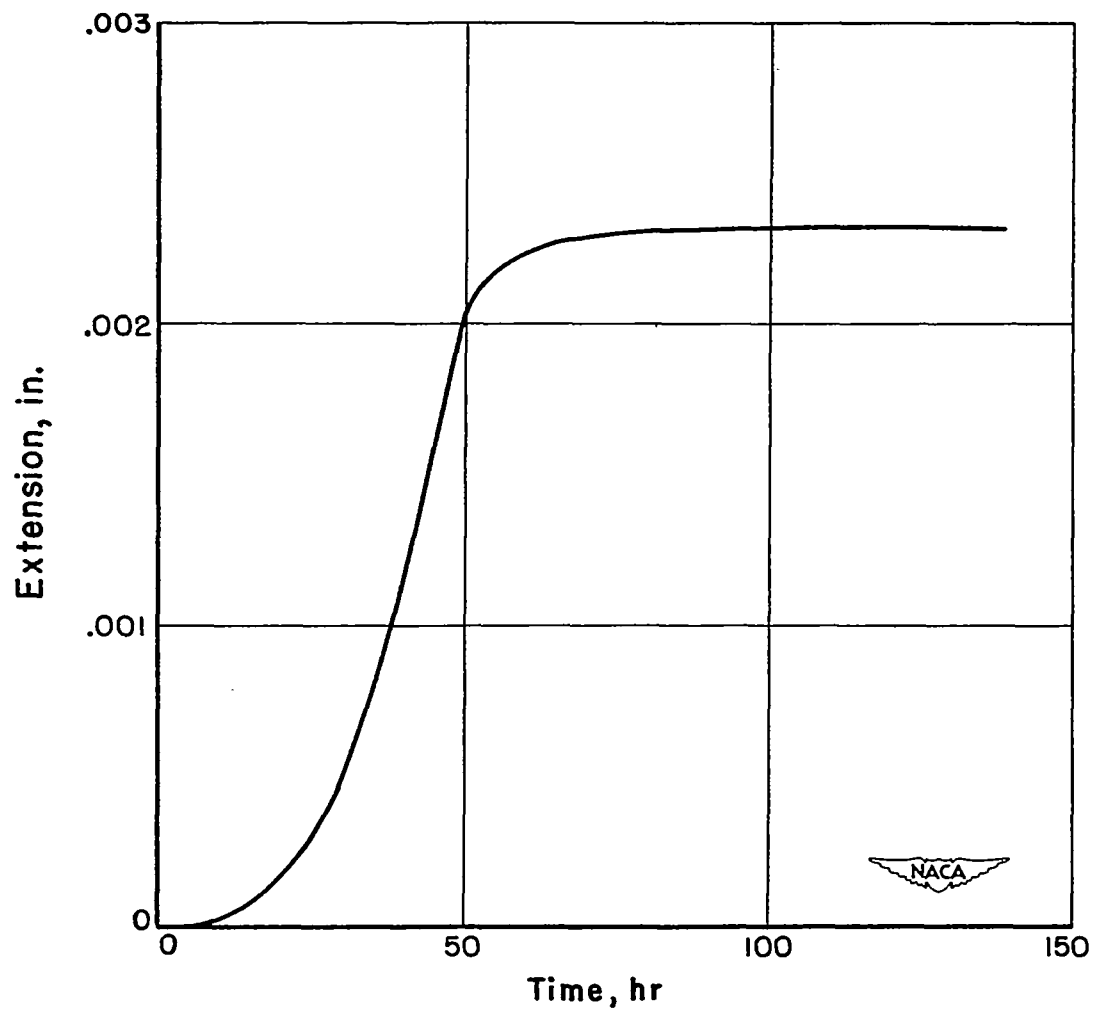resolved shear stress, 600 grams per square millimeter.

Figure 9.- Time-deformation curve for a single iron crystal at a stress
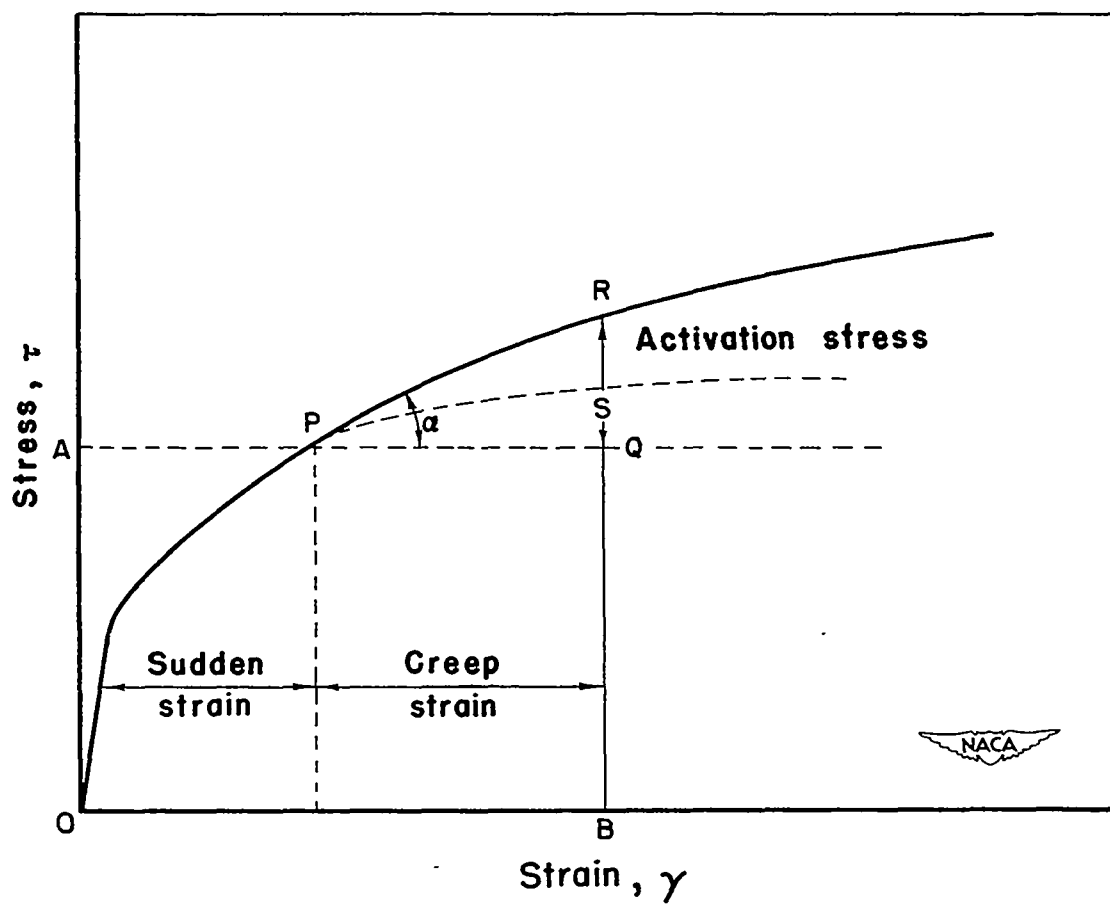above yield point.   (From Gensamer and Mehl, reference 78.)

Figure 10.- Stress-strain curve showing mechanism of transient creep.
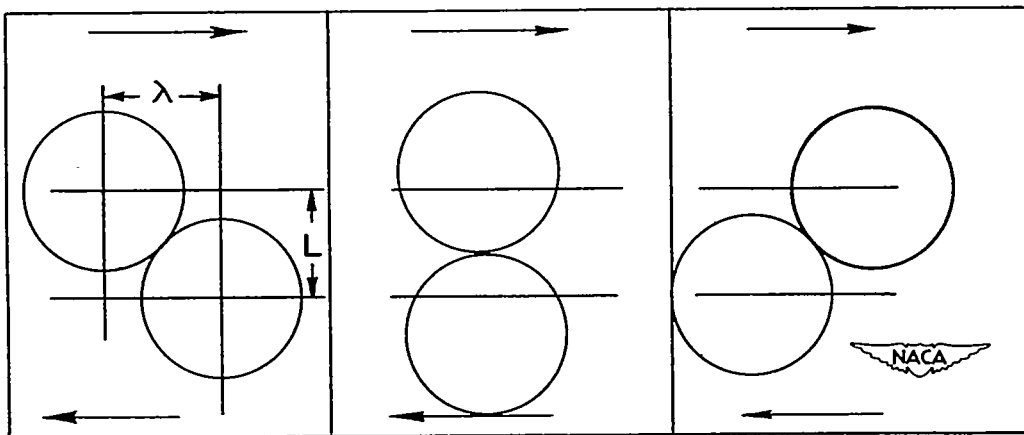(From Orowan, reference 155.)

Figure 11.- Molecular shear process in liquid.   (From Kauzmann, reference 112.)